Experiment 3

Analysis of a Freefalling Body

3.1 Objectives

- Verify how the distance of a freely-falling body varies with time.
- Investigate whether the velocity of a freely-falling body increases linearly with time.
- Calculate a value for g, the acceleration due to gravity.

3.2 Introduction

Everyday, you experience gravity. This happens because the Earth is so massive, it pulls us down and keeps us on the ground. But happens when we drop something? We notice that as this thing falls to the Earth, it moves faster and faster until it hits the ground. From this we can tell that gravity is **accelerating** the object the entire time the object is in freefall. Today, we will measure how much gravity actually accelerates this object by using the Behr Freefall apparatus and your mathematical skills.

3.3 Key Concepts

As always, you can find a summary on-line at Hyperphysics[1]. Look for keywords: gravity, velocity, and acceleration

[1] http://hyperphysics.phy-astr.gsu.edu/hbase/hph.html

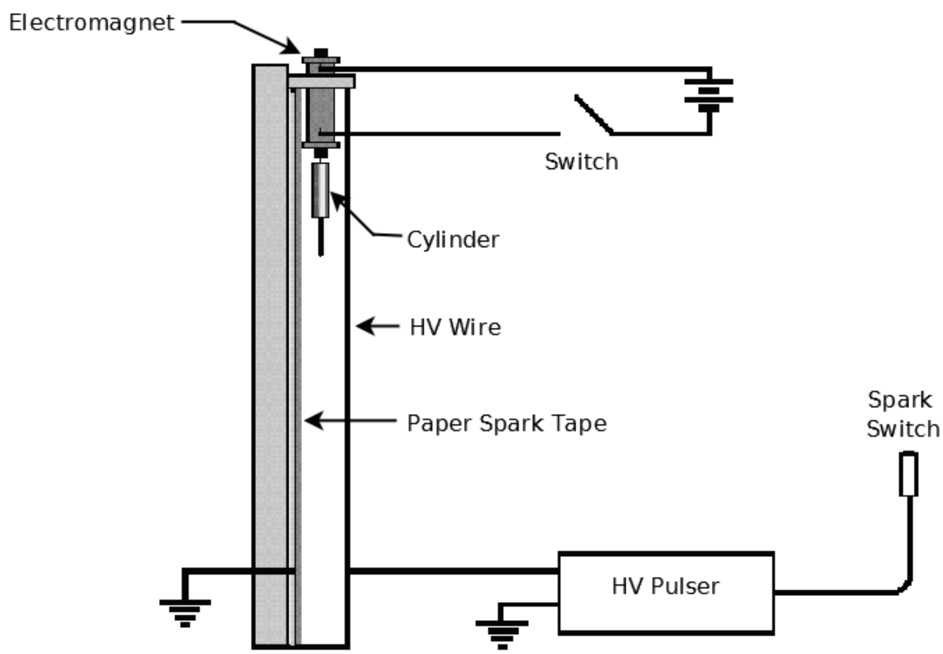

Figure 3.1: Schematic of the Behr Freefall Apparatus.

3.4 Apparatus

A Behr Free-Fall Apparatus and Spark Timing System will be used in this experiment. A schematic representation of the apparatus is shown above in Fig. 3.1 and a digital photograph of the apparatus is shown in Fig. 3.2 on the following page.

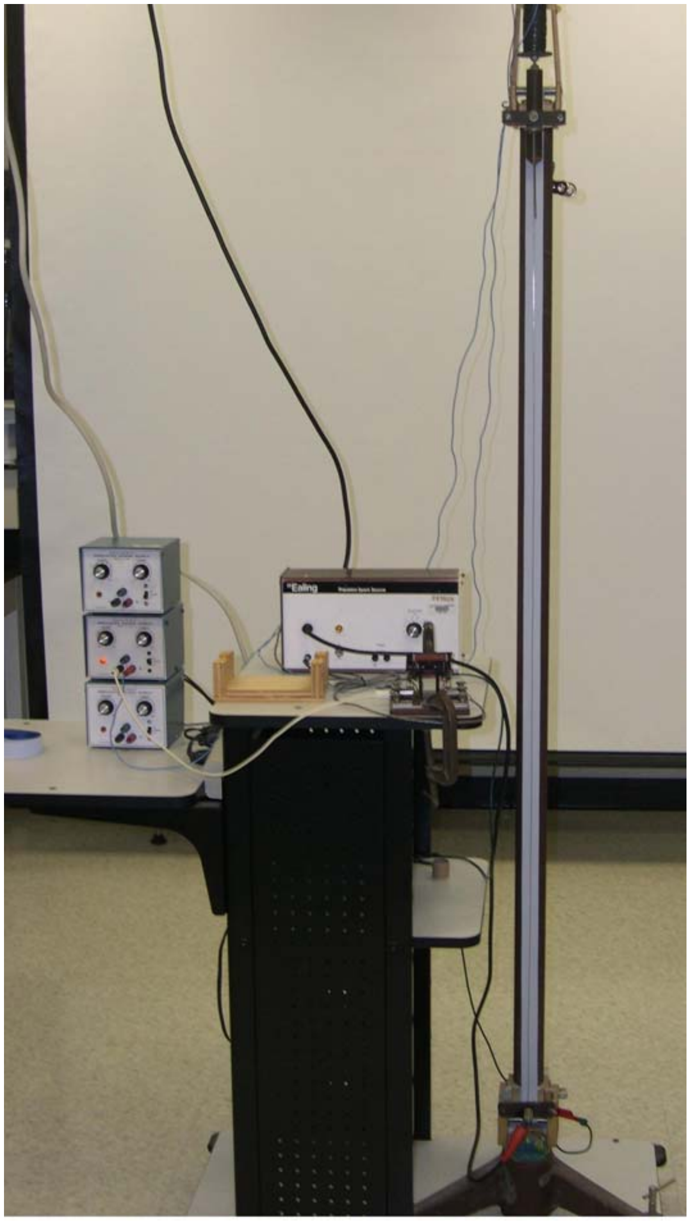

Figure 3.2: Schematic of the Behr Freefall Apparatus.

3.5 Theory

In this experiment a cylinder is dropped and a record of its free fall is made. Before the measurement, the cylinder is suspended at the top of the stand with the help of an electromagnet. When the electromagnet is turned off, the cylinder is released and starts to fall. Simultaneously, the spark timer starts to send high-voltage pulses through two wires which are stretched along the cylinder's path. At the time of each pulse a spark goes through the wires and the cylinder, leaving a mark on the special paper tape that lies between the cylinder and one of the wires. The **time interval** between two adjacent sparks is constant and is denoted by the Greek letter tau "τ". $\tau = 1/60$ of a second. Measuring the distances between any two marks, Δy, and knowing the time interval between the corresponding sparks, Δt, it is possible to calculate the **average** velocity during this interval using the formula

$$v = \frac{\Delta y}{\Delta t} \tag{3.1}$$

If Δt is small enough, we can assume that the velocity at any instant within this interval is approximately equal to this average velocity. In the case where acceleration is constant, the **instantaneous velocity** at the middle of the time interval Δt is exactly equally to the average velocity of the object during the time interval Δt.

In general, for the motion of a body with a constant acceleration a, the velocity v is given by the equation

$$v = at + v_0 \tag{3.2}$$

where v_0 is the velocity of the cylinder at $t = 0$. Since in our case the body is falling freely,

$$a = -g \tag{3.3}$$

where $g = 9.81\mathrm{m/s}^2$ is the magnitude of the acceleration due to gravity. The negative sign in front of g is to indicate that the direction of the acceleration is in the negative direction (i.e. downward). Therefore it follows from Eq. 3.2 that for a freely-falling body

$$v = v_0 - gt \tag{3.4}$$

Thus g can be determined from a plot of v vs. t since the slope of any velocity versus time graph is just the acceleration. The obtained value of g can then be compared with the known value of the acceleration due to gravity. The position of the cylinder, y, as a function of time is given by the standard equation for an object that is undergoing constant acceleration. If at time $t = 0$ the object has height y_0 and velocity in the vertical direction v_0, then this equation looks like

$$y = y_0 + v_0 t - \frac{1}{2} g t^2 \tag{3.5}$$

3.6 In today's lab

At the start of today's lab, your instructor will demonstrate the operation of the Behr Freefall apparatus. In the interest of saving time, you will be supplied with a shock tape from the Behr freefall apparatus. On this tape, you will then measure the distance between the "dots," and using the given τ, you will calculate the value of acceleration for g.

3.7 Equipment

- Behr Freefall Apparatus
- Shock tape
- Meter stick

3.8 Procedure

1. Secure both ends of the shock tape to the desk using masking tape, making sure that the shock tape is as flat as possible.
2. The "bottom" of the tape is defined as the end of the tape with the largest, bold black dot, also where the dots become farther apart. Starting from the **third** dot from the bottom, label each successive point from #25 to #1 in **descending** order. It is ok if you have unused marks left over. Point #25 will now be defined at $y = 0$ and point #1 will be defined at $t = 0$.

3. Using your meter stick, measure each point's distance (in cm) from $y = 0$ (point #25) and write it down on the shock tape.

4. Input your measured distances into excel paying special attention to which point number you are putting the distance into. Also input a reasonable uncertainty for your measurement in excel.

5. Input the correct times for each point using the given value of τ.

6. Calculate the instantaneous velocity v_i for each point y_i. Here we have that $v = \frac{\Delta y}{\Delta t}$. Δy for each point i is defined as $\Delta y = y_{i+1} - y_{i-1}$ and Δt is likewise defined as $\Delta t = t_{i+1} - t_{i-1}$. For example, we can see that $v_2 = \frac{y_3 - y_1}{t_3 - t_1}$. Note that Δt will be the same value for each point, which happens to just be 2τ. Please include at least 1 hand calculation for these values.

7. Now calculate the uncertainty for your velocity at each point using the equation $\delta v_i = \frac{\delta y}{\tau}$.

8. Transfer your data columns for "Time", "y", and "v" into Kaleido-Graph.

9. Make a graph for y vs. t. You do not need a best fit line or error bars for this plot.

10. Make a graph for v vs. t. Be sure to include a best fit line and error bars for v. Be sure to write in plot comments for both of your plots.

3.9 Error Calculation

For each measured y_i you assign an error based on how accurately you can measure that point. This error is called δy. This error determines all other errors in this lab. For this lab and for the following formulae it is assumed that the error in τ and m are zero.

There error in Δy at each point i is the same and is given by

$$\delta(\Delta y) = 2\delta y \tag{3.6}$$

The error in the speed at each point i is

$$\delta(v_y) = v_y \frac{\delta(\Delta y)}{\Delta y} = v_y \frac{2\delta y}{\Delta t} = \frac{(\Delta y) 2\delta y}{2\tau(\Delta y)} = \frac{\delta y}{\tau} \tag{3.7}$$

3.10 Checklist

1. Your spreadsheet and formula view.
2. Sample calculations.
3. Plot of the height vs. time Graph I.
4. Plot of velocity vs. time Graph II.
5. Interpretation of the two plots.
6. Answered questions.
7. One member of each group should turn in your spark tape record of the free-fall.
8. If your lab instructor tells you that you will need to use this data for next the next lab, make sure you save your spreadsheet.

3.11 Questions

1. What is the y-intercept determined from your Graph II (or from the equation of its best-fit line)? What does it mean?

2. Using equation 3.4 and the equation of the best fit line from Graph II, calculate the time at which $v = 0\ cm/s$. Explain this result.

3. What is your value of the gravitational acceleration in cm/s^2 determined from the slope from graph II? Is this value compatible with the accepted value of the gravitational acceleration? If not suggest a possible source of error (**NEVER** just suggest “human error” or a “mistake”).

4. When the initial velocity is zero, what would you plot to make graph I linear: y^2 vs. t, y^2 vs. t^2, or y vs. t^2? As always, justify your response.